Lecture Notes: November 18, 2010

TIME AND UNCERTAINTY: FUTURES MARKETS

Gerard says: theory's in the math. The rest is interpretation.

- make the markets for goods over time look just like those in the general equilibrium model and the same formal results will follow. You'll be able to establish an intertemporal equilibrium, and intertemporally efficient allocation. All that remains is to interpret what economic institutions it requires for intertemporal goods allocation to look like the general equilibrium model. Futures markets; Chicago Board of Trade.

- make the markets for goods under uncertainty look just like those in the general equilibrium model and the same formal results will follow. You'll be able to establish an equilibrium for goods across uncertain events, and an efficient allocation of risk bearing. All that remains is to interpret what economic institutions it requires.

For the market:

- all economically significant scarce resources are traded in the market; goods distinct from one another in production or consumption are distinct co-ordinates in N-dimensional commodity space. A single market date where all futures contracts are traded.
- there is a single market date at which all supplies and demands are expressed and equated. Budget constraints and firm profits are expressed effective with this date. Thus the complete future is collapsed --- for market purposes into the single market date.

For the firm:

- there is a single scalar maximand, profit.
- all economically relevant production possibilities are fully expressed in the firm technology set.

For the household:

- there is a single maximand, $\succsim_i$ or equivalently the scalar $u^i(\cdot)$.
- there is a single scalar budget constraint.

For the economy:

- firm profits are distributed to households. Walras' Law holds.

<u>Time, Futures Markets</u>

A commodity is characterized

by *what* it is (its description), and

by *where* it is available (its location), and

by *when* it is available (its date).

There are actively traded goods for all dates --- if a good will be available at a particular date in the future, futures contracts for the good deliverable at that date are traded in the market at the market date.

N finite. N includes as a separate count every good, at every location where it is deliverable, and at every date at which it is deliverable.

There is only a single meeting of the market. equilibrium: for all dated goods. Each household has only a single budget constraint representing receipts and expenditures at all dates from the present to the <u>finite horizon</u>. Firms have only a single calculation of profit representing the net return on receipts for outputs and expenditures for inputs over all dates from the present to the finite horizon. All receipts and expenditures for spot (current) goods and future deliveries are evaluated at the single market date. The distinctions between income and wealth, firm profits and firm value, are eliminated.

p_i = present discounted value of commodity i discounted from the delivery date to the market date.

Costs are incurred, revenues received, accounts debited and credited at the market date, long prior to delivery.

Household endowment $r^h \equiv (r_1^h, r_2^h, ..., r_N^h)$: present and future goods.

Household consumption $x^h \equiv (x_1^h, x_2^h, ..., x_N^h)$: Each co-ordinate represents dated planned consumption of a particular good.

j's production y^j : dated plan for inputs and outputs at a sequence of dates.

Input and output prices are discounted values, discounted to the market date. j's profit is $\pi^j(p) = \max_{y \in Y^j} p \cdot y = p \cdot S^j(p)$. $\pi^j(p)$ is the sum evaluated at the market date, over all dates from the present through the time horizon of the (present discounted) value of outputs less the (present discounted) value of inputs. Maximizing firm (discounted) profit, maximizing shareholder wealth, and maximizing firm (stock market) value are identical.

$\succsim_h$, u^h represent preferences on time dated streams of consumption from the present through the future until the horizon.

$M^h(p) = p \cdot r^h + \sum_{j \in F} \alpha^{hj} \pi^j(p)$. h chooses a consumption plan for the present through the horizon to optimize a planned program of consumption evaluated by h's preferences for consumption across goods and time, subject to the budget constraint that the present discounted value of the consumption plan is bounded above by the present discounted value of endowment plus the value of firm ownership.

The futures markets here perform the functions both of goods markets and of capital markets. Investment and saving are arranged through futures markets. All trade takes place prior to consumption or production. Too many markets.

A Sequence Economy

At each date there are spot markets for active trade in goods deliverable at that date. There are financial markets in debt instruments, borrowing and lending into the future. Firms and households have perfect foresight concerning the prices prevailing in the future. Budget constraint at each date: sales of goods and debt (borrowing) must finance purchases. Sequence economy model with complete debt markets corresponds to the concept of a perfect capital market. Requires foresight on spot prices.

Uncertainty - Arrow(-Debreu) Contingent Commodity Markets

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A commodity is now characterized
by *what* it is (its description), and
by *where* it is available (its location),
by when it is available (its date), and
by it's *state of the world* (the uncertain event in which it is deliverable).
Arrow (1954, 1962)

N finite: number of possible uncertain events is finite at every date (& time is finite).

Contingent commodity: specific good *deliverable if a specified event occurs*.
An event tree.

y^j, firm j's production plan, in this setting, is chosen to maximize the value of $p\cdot y$ for y in Y^j. Firm output in any date/event is known with certainty and reliability. the firm does not need a forecast nor an attitude toward risk: households have attitudes toward risk, which get priced into the market for contingent commodities. Firm j chooses y^j (a dated contingent commodity production plan) to maximize $\pi^j(p) = \max_{y\in Y^j} p\cdot y = p\cdot S^j(p)$ = stock market value of the firm.

$$M^h(p) = p\cdot r^h + \sum_{j\in F} \alpha^{hj}\pi^j(p).$$

Household h's consumption vector x^h represents a state contingent dated list of projected consumptions. Household needs attitude toward risk and judgment of likelihood of realization of events. May be expected utility maximizer; optimizes utility of portfolio.

Most contingent commodity contracts expire without being executed by delivery. No reopening of markets.

Equilibrium allocation of risky assets is Pareto efficient relative to $\succsim_h$. Given the endowments r^h and available technologies Y^j, there is no attainable reallocation of inputs to firms j or of contingent commodity outputs to households h that would move some household h higher in its ranking of portfolios, $\succsim_h$, without moving some other household h' lower in its ranking of portfolios, $\succsim_{h'}$. This means that the allocation of risk bearing among households is Pareto efficient, an efficient allocation of risk bearing. There is no rearrangement of the risky assets, the contingent commodities, among households that would be Pareto improving in terms of household portfolio preferences. Ex ante efficiency distinct from ex post efficiency.

Described in the literature as "a full set of Arrow-Debreu futures contracts" or "a full set of Arrow-Debreu contingent commodities". <u>Too many markets.</u> See literature on GEI, general equilibrium with incomplete markets.

Uncertainty - Arrow Securities Markets

Too many markets.

Arrow insurance contract: Suppose there is a 'money' or numeraire in which we can describe a payment of generalized purchasing power. For each date event pair, t,s , the contract c(t,s) pays one unit of purchasing power if event s occurs at date t, and nil otherwise. Then instead of a full set of contingent commodity markets, we can use a mix of insurance contracts and spot markets (markets for actual goods deliverable in the current period) to achieve the same allocation as available in the contingent commodity equilibrium.

Correctly foreseen state-contingent goods prices.

Formal equivalence

- Contingent commodity markets:

  the market meets once for all time and a very large number of contingent commodities are traded; most do not result in delivery of actual goods.

- Securities markets for Arrow insurance contracts payable in abstract purchasing power:

  the securities market meets once; goods markets re-open at each date for spot trade. Most securities do not result in actual payment.

Replace the full set of contingent commodity markets with a much smaller number of markets.

Ex ante and ex post efficiency under uncertainty

In the equilibrium of an Arrow-Debreu model with a full set of contingent commodity futures markets the *ex ante* allocation of risk bearing is Pareto efficient. There is no claim that the resulting allocation of resources *ex post* is efficient (with the benefit of 20-20 hindsight).

Consider households i and j in a 2-period model. Each has an expected utility function of the form

$$Eu^h = u^h (x^h_o) + \theta^h_1 u^h(x^h_1) + \theta^h_2 u^h(x^h_2)$$

where $u^h (x^h_o)$ is h's utility of period 0 consumption, θ^h_1 is h's subjective probability of event 1 in period 1, x^h_1 is h's contingent consumption at date 1 state 1, θ^h_2 is h's subjective probability of state 2 date 1, x^h_2 is h's contingent consumption at date 1 state 2 (this is crummy notation, but avoids more than one subscript at a time). Prices are p_o for date 0 consumption, p_1 for date 1 state 1 contingent consumption, and p_2 for date 1 state 2 contingent consumption. Then competitive equilibrium on the *ex ante* contingent commodity market (and an efficient allocation of risk bearing) requires

$$\frac{p_o}{p_1} = \frac{u^{i\prime}(x_o^i)}{\theta_1^i u^{i\prime}(x_1^i)} = \frac{u^{j\prime}(x_o^j)}{\theta_1^j u^{j\prime}(x_1^j)}$$

But *ex post* efficient allocation of resources requires

$$\frac{u^{i\prime}(x_o^i)}{u^{i\prime}(x_1^i)} = \frac{u^{j\prime}(x_o^j)}{u^{j\prime}(x_1^j)} \ .$$

But these two conditions (assuming unique interior solutions) will only be consistent with one another if

$\theta_1^i = \theta_1^j$, and there is no particular reason to expect this equality.